

OU-TAP 178

## Vacuum selection by inflation as the origin of the dark energy

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### Abstract

I propose a new mechanism to account for the observed tiny but finite dark energy in terms of a non-Abelian Higgs theory, which has infinitely many perturbative vacua characterized by a winding number, in the framework of inflationary cosmology. Inflation homogenizes field configuration and practically realizes a perturbative vacuum with vanishing winding number, which is expressed by a superposition of eigenstates of the Hamiltonian with different vacuum energy density. As a result, we naturally find a nonvanishing vacuum energy density with fairly large probability, under the assumption that the cosmological constant vanishes in some vacuum state. Since the predicted magnitude of dark energy is exponentially suppressed by the instanton action, we can fit observation without introducing any tiny parameters.

One of the greatest mysteries of contemporary cosmology is the origin of dark energy or vacuum-like component of cosmic energy indicated by the analysis of type Ia supernovae observations<sup>1–3</sup>, whose magnitude is some 120 orders smaller than its natural value, the Planck density<sup>4</sup>. In this essay I propose a novel mechanism to account for the observed tiny but finite dark energy without introducing any small numbers in the framework of inflationary cosmology<sup>5</sup> making use of a non-Abelian Higgs theory, which is a part of typical unified theories of elementary interactions, with infinitely many perturbative vacua characterized by winding numbers. In the proposed mechanism inflationary expansion in the early universe plays an essential role in realizing the appropriate quantum state. So let us first recall its cosmological implications.

Inflation stretches preexistent inhomogeneities and yields an extremely large smooth domain in which the current Hubble volume is contained. It also provides a generation mechanism of density fluctuations which originate in the quantum nature of the inflation-driving scalar field<sup>6–8</sup>. In calculating perturbations, we consider the field to be in the vacuum state and calculate its zero-point quantum fluctuations. This treatment should be an extremely good approximation, if not exact, as long as we focus on perturbation within currently observable Hubble volume. This is because, even if the scalar field was in some excited state initially, subsequent inflation would exponentially dilute the preexistent quanta. Indeed the trace of such an initial excitation could be seen only if an observation was made across an exponentially large region far beyond the current Hubble horizon, which is of course infeasible. Thus for all the practical purposes we can regard that the quantum state of the scalar field is no different from the vacuum state in this context. That is to say, the vacuum state is dynamically selected as a result of cosmological inflation.

Next let us consider a non-Abelian gauge theory with the gauge group  $G$  which has infinitely many perturbative vacuum states classified by the winding number  $n$ . Most gauge groups used in grand unified theories such as  $G = \text{SU}(N)$ ,  $\text{SO}(N)$ , and  $\text{Sp}(N)$  with  $N \geq 2$  possess this property. For definiteness, however, we concentrate on the simplest theory with this property,  $G = \text{SU}(2)$ , below. In these theories perturbative vacuum states with different winding numbers cannot be transformed from each other by a continuous gauge transformation<sup>9</sup> and there is an energy barrier between them. Quantum transition, however, is possible and described by an instanton solution which is an Euclidean solution connecting two adjacent perturbative vacua with a finite Euclidean action  $S_0 = 8\pi^2/g^2$  with  $g$  being the gauge coupling constant<sup>10–11</sup>. As a result the true vacuum state is expressed by a superposition of perturbative vacua,  $|n\rangle$ , as

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{in\theta} |n\rangle, \quad (1)$$

where  $\theta$  is a real parameter<sup>9,12</sup>. One can easily find that this state is a real eigenstate of the Hamiltonian  $\mathcal{H}$  by calculating the amplitude  $\langle\theta'|e^{-\mathcal{H}T}|\theta\rangle$  with the dilute instanton approximation<sup>12</sup>, namely, adding contributions of  $m$  instantons and  $\bar{m}$  anti-instantons keeping in mind that each (anti-)instanton changes the winding number by  $(-1)1$ .

$$\begin{aligned} \langle\theta'|e^{-\mathcal{H}T}|\theta\rangle &= \sum_{n,n'} \langle n' | e^{-\mathcal{H}T} | n \rangle e^{in\theta - in'\theta'} = \sum_{n,n'} \sum_{m,\bar{m}} \frac{1}{m!} \frac{1}{\bar{m}!} \left( K V T e^{-S_0} \right)^{m+\bar{m}} \delta_{m-\bar{m},n-n'} e^{in\theta - in'\theta'} \quad (2) \\ &= \sum_{n,n'} e^{in(\theta-\theta')} \sum_{m,\bar{m}} \frac{1}{m!} \frac{1}{\bar{m}!} \left( K V T e^{-S_0} \right)^{m+\bar{m}} e^{-i(m-\bar{m})\theta} = \exp \left( 2 K V T e^{-S_0} \cos \theta \right) \delta(\theta - \theta'), \end{aligned}$$

Here  $K V T e^{-S_0}$  represents contribution of a single instanton or anti-instanton where  $K$  is a positive constant and  $V T$  represents spacetime volume.

Then the above equality (2) clearly shows that each  $\theta$ -vacuum  $|\theta\rangle$  has a different energy density than the perturbative vacuum  $|n\rangle$  by  $\Delta\rho = -2Ke^{-S_0} \cos\theta$ . Apparently the  $\theta = 0$  vacuum has the lowest energy, but one cannot conclude that this is the only vacuum state, because the other  $\theta$ -vacua are also stable against gauge-invariant perturbations<sup>9,12</sup>. In fact, the factor  $K$  is divergent due to the contribution of arbitrary large instantons in pure gauge theory<sup>13</sup>. In order to obtain a physical cutoff scale, let us introduce an SU(2) doublet scalar field  $\Phi$  with a potential  $V[\Phi] = \lambda(|\Phi|^2 - M^2/2)^2/2$  following 't Hooft<sup>13</sup>. For  $M \neq 0$ , instead of the instanton solution of pure gauge theory we should use a constrained instanton solution which also connects adjacent perturbative vacua with a finite Euclidean action<sup>14,15</sup>. Then the prefactor is given by<sup>16</sup>  $K \cong (8\pi/g^2)^4 M^4/2$  and the expectation value of the vacuum energy density in perturbative vacuum  $|n\rangle$  and that in  $\theta$ -vacuum  $|\theta\rangle$  are given by

$$\langle n|\mathcal{H}|n\rangle = \left(\frac{8\pi}{g^2}\right)^4 M^4 e^{-\frac{8\pi^2}{g^2}} + \rho_v(\theta = 0), \quad (3)$$

$$\rho_v(\theta) \equiv \langle \theta|\mathcal{H}|\theta\rangle = \left(\frac{8\pi}{g^2}\right)^4 M^4 e^{-\frac{8\pi^2}{g^2}} (1 - \cos\theta) + \rho_v(\theta = 0) \equiv \rho_d(1 - \cos\theta) + \rho_{v0}, \quad (4)$$

respectively. Here  $\rho_d \equiv (8\pi/g^2)^4 M^4 e^{-\frac{8\pi^2}{g^2}}$ , and  $\rho_v(\theta = 0) \equiv \rho_{v0}$  is the vacuum energy density in the state  $|\theta = 0\rangle$ . Since different vacuum states have different vacuum energy density, they behave differently in the presence of gravity. Hence any good solution to the conventional cosmological constant problem, namely why the vacuum energy density vanishes, should specify at which vacuum state the cosmological constant vanishes. Unfortunately, since there is no completely satisfactory solution to this long-standing problem yet, let us simply normalize the vacuum energy density to vanish in the CP-symmetric  $\theta = 0$  vacuum state and set  $\rho_{v0} = 0$  for the moment just for definiteness<sup>†</sup>. As will be seen below, however, this choice is not essential and our mechanism can work even if the vacuum energy density vanishes at some other vacuum except at  $|\theta = \pi\rangle$ .

Now let us consider what quantum state is chosen by cosmological inflation in this non-Abelian-Higgs system. Let us assume the energy scale of the Higgs field,  $M$ , is much larger than the Hubble parameter during inflation,  $H$ , which is shown to be the case later. As in the case of singlet scalar field, sufficiently long inflation would homogenize the Higgs field configuration over an exponentially large domain with  $|\Phi| = M/\sqrt{2}$ . This in turn means that the state with vanishing winding number  $|n = 0\rangle$  is practically realized as long as we concentrate on the scales within the current Hubble volume, because this is the only state with homogeneous scalar field configuration among many possible vacuum states, perturbative  $|n\rangle$  or real  $|\theta\rangle$ . In terms of the energy eigenstates this state is expanded as

$$|n = 0\rangle = \int_0^{2\pi} \frac{d\theta}{2\pi} |\theta\rangle, \quad (5)$$

which consists of superposition of all possible real vacuum state  $|\theta\rangle$  with equal weight.

While we can calculate the expectation value of vacuum energy density in this state as (3), it is more appropriate to discuss the probability distribution function (PDF) of vacuum energy

<sup>†</sup>We point out that the wormhole mechanism<sup>17</sup> is an example of proposed solutions to the conventional cosmological constant problem which predicts the vacuum energy density vanishes at  $\theta = 0$  vacuum<sup>18</sup>.

because this state consists of superposition of energy eigenstates with different energy density. Using (5) we can easily calculate the PDF of vacuum energy density,  $P(\rho_v)$ , in this state as

$$P(\rho_v) = \left\langle \delta(\rho_v - \rho_v(\theta)) \right\rangle_\theta = \int_0^{2\pi} \frac{d\theta}{2\pi} \delta(\rho_v - \rho_d(1 - \cos \theta)) \\ = \begin{cases} \frac{1}{\pi(2\rho_v\rho_d - \rho_v^2)^{1/2}} & \text{for } 0 \leq \rho_v \leq 2\rho_d, \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

We can also calculate the cumulative probability distribution function to find  $\rho_v \geq \rho$  as

$$P(\rho_v \geq \rho) = \int_\rho^{2\rho_d} P(\rho) d\rho = \frac{1}{2} - \frac{1}{\pi} \arcsin \left( \frac{\rho}{\rho_d} - 1 \right), \quad (7)$$

for  $0 \leq \rho \leq 2\rho_d$ . Figures 1 and 2 depict these PDFs (6) and (7), respectively. As is seen there  $P(\rho_v)$  is sharply peaked at  $\rho_v = 0$  corresponding to  $\theta = 0$  and  $\rho_v = 2\rho_d$  corresponding to  $\theta = \pi$  and the probability to find  $\rho_v \approx 2\rho_d$  is fairly large as seen in Figure 2.

Thus, if the perturbative vacuum  $|n=0\rangle$  is stable in the cosmological time scale, the PDF of the vacuum energy density is still given by (6) today in our universe which experienced inflation. Hence it is by no means surprising that we observe a finite dark energy around  $2\rho_d$  today.

We can easily match the predicted value,  $\approx 2\rho_d$ , with the observed one,  $10^{-120} M_G^4$ , by appropriately choosing  $M$  and  $g$ , where  $M_G$  is the reduced Planck scale. We also demand that the tunneling rate from  $|n=0\rangle$ ,  $\Gamma \approx K e^{-\frac{16\pi^2}{g^2}}$ , is small enough to guarantee that there is no transition within the current Hubble volume,  $H_0^{-3}$ , in the cosmic age,  $\Gamma H_0^{-4} \lesssim 1$ . Then we obtain

$$\pi/\alpha + 2 \ln \alpha = 60 \ln 10 + 2 \ln(M/M_G), \quad \text{and} \quad M \gtrsim \alpha M_G, \quad (8)$$

where  $\alpha \equiv g^2/(4\pi)$  is the coupling strength at the energy scale  $M/\sqrt{2}$ . If the above inequality is marginally satisfied, we find  $\alpha = 1/44.4$  and  $M = 5 \times 10^{16} \text{ GeV}$ . If, on the other hand, we take  $M = M_G$  so that the cutoff scale of instanton is identical to the presumed field-theory cutoff, we find  $\alpha = 1/47$ .

Several comments are in order. (i) Since the scale of the Higgs field  $M$  turns out to be much larger than the Hubble parameter during inflation, which is constrained as<sup>19–21</sup>  $H/(2\pi) \lesssim 4 \times 10^{13} \text{ GeV}$ , quantum fluctuation does not affect the realization of the state  $|n=0\rangle$ . (ii) Since the reheat temperature after inflation is much smaller than  $M$ , there is no thermal transition to destabilize the state  $|n=0\rangle$  after inflation. (iii) We may find CP nonconserving value,  $\theta \neq 0, \pi$ . But this would not cause phenomenological troubles because our  $\theta$  is not directly related with that of strong interaction. (iv) It is not mandatory to assume that the vacuum energy density vanishes at the state  $|\theta=0\rangle$ . One may take other vacuum state to normalize the origin of the vacuum energy density. For example, if one assumes that  $\rho_v$  vanishes at  $|n\rangle$  or  $|\theta=\pi/2\rangle$ ,  $P(\rho_v)$  will be peaked at  $\rho_v = \pm\rho_d$  instead, which we may interpret that we live in a universe with  $\rho_v \approx \rho_d$ . Then the values of the model parameters should be slightly changed accordingly.

In conclusion, we have reached a striking conclusion that the observed tiny dark energy can be realized by the interplay of high energy physics between grand unification and Planck scales and inflationary cosmology. Furthermore by virtue of the exponential dependence of the dark energy on the action of instanton, we can reproduce its observed value without introducing any tiny numbers.

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FIGURES

Figure 1

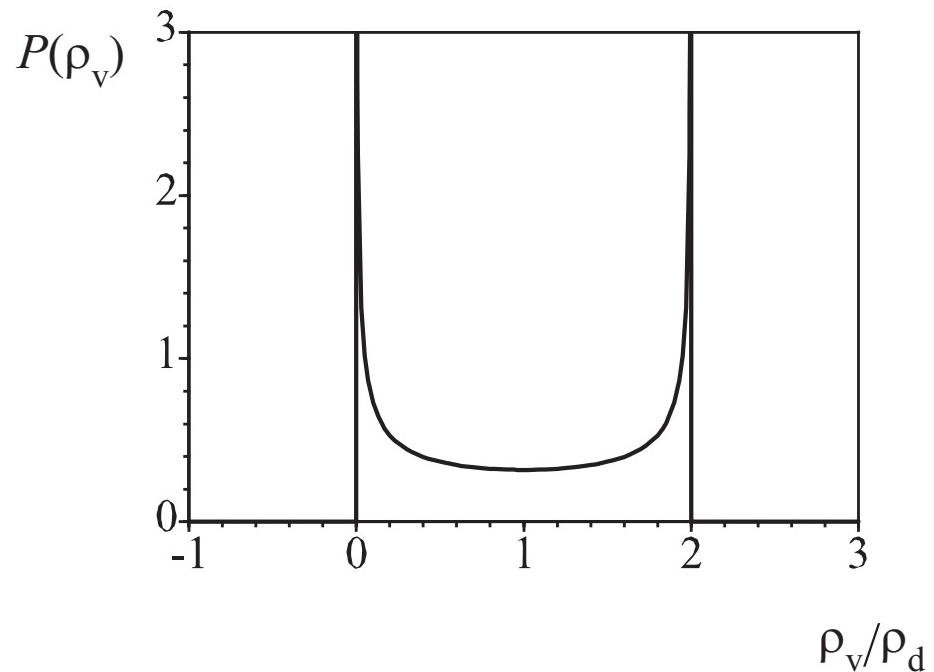


Figure 2

